Math 3150 Exam 3A, April 2026, 13 problems, 75 minutes   Name: Answers

Read these instructions:

- Leaving the testing room results in a new exam given for the unfinished problems.
- Two detached sheets of notes allowed.
- No electronics.
- Raise your hand for questions or more paper.

Problem 1. Write the following statements in terms of the letters m = "Jo is a math major", c = "Jo is a CS major", d = "Jo is a DS major" and the symbols $\neg, \wedge, \vee, \oplus$.

+2 **Part A.** "Jo is a math major and a DS major but not a CS major." $m \wedge d \wedge \neg c$

+2 **Part B.** "Jo is neither a math major nor a CS major." $\neg(m \vee c) \equiv \neg m \wedge \neg c$

+2 **Part C.** "Jo is not a CS major." $\neg c$

+2 **Problem 2A.** State the **converse** of "If it walks like a duck, then it talks like a duck" in English.

If it talks like a duck, then it walks like a duck

+2 **Problem 2B.** State the **contrapositive** of "If it walks like a duck, then it talks like a duck" in English.

If it does not talk like a duck, then it does not walk like a duck.

+5 **Problem 3.** Find a disjunctive normal form for $(p \vee q) \Rightarrow p$.

p	q	$p \vee q$	$(p \vee q) \Rightarrow p$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	T

So... $(p \vee q) \Rightarrow p \equiv (p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge \neg q)$

other answers: $\neg q \vee p$

+5 **Problem 4.** Is $(p \Rightarrow q) \wedge q$ logically equivalent to $q \Rightarrow p$? Explain.

p	q	$p \Rightarrow q$	$(p \Rightarrow q) \wedge q$	$q \Rightarrow p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

Final two columns are different, so not logically equivalent

+2 **Problem 5.** Let $T(x)$ ="x has thorns", $P(x)$ ="x has petals", and R be the set of all roses. Write "Some rose has thorns but every rose has petals" in terms of x, $T(x)$, $P(x)$, R, $\forall$, $\exists$, $\in$, :, [,], $\wedge$, $\vee$, $\neg$, $\Rightarrow$.

$[\exists x \in R : T(x)] \wedge [\forall x \in R : P(x)]$

Problem 6. Let $B(x)$ = "x has at least one bird", $C(x)$ = "x has at least one cat", $D(x)$ = "x has at least one dog", and S be the set of students in this class. Formalize (a)-(c) via the symbols $S, x, B(x), C(x), D(x), \wedge, \vee, \neg, \Rightarrow, \forall, \exists, :$ (colon), $\in, [,]$.

+2 **Part A.** Some student in this class has at least one bird and at least one dog.

$\exists x \in S : B(x) \wedge D(x)$

+2 **Part B.** Every student in this class has at least one bird, at least one cat, or at least one dog.

$\forall x \in S : B(x) \vee C(x) \vee D(x)$

+2 **Part C.** Some student in this class has at least two types of animals out of birds, cats, and dogs.

$\exists x \in S : [C(x) \wedge B(x)] \vee [B(x) \wedge D(x)] \vee [C(x) \wedge D(x)]$.

+2 **Problem 7.** Which option below is equivalent to the **negation** of "Every teapot is short and stout"?

Some not short or not stout.

(a) Some teapot is not short and not stout.
(b) Some teapot is not short or not stout.
(c) Every teapot is not short and not stout.
(d) Every teapot is not short or not stout.

Problem 8. Mark each item as true or false. Justify your answers:

+2 **Part A.** $\forall x \in \mathbb{R} : x^2 \geq x$ False: let $x = \frac{1}{2}$. Then $x^2 = \frac{1}{4} < x = \frac{1}{2}$.

+2 **Part B.** $\exists x \in \mathbb{N} : x^2 < x$ False: $x^2 \geqslant x$ for all natural #'s.

+2 **Part C.** $\forall x \in \mathbb{R} : [\exists y \in \mathbb{R} : x^2 + y = 0]$ True: let $y = -x^2$

+2 **Part D.** $\forall y \in \mathbb{R} : [\exists x \in \mathbb{R} : x^2 + y = 0]$ False: if $y = +1$, then $x^2 + 1 = 0 \Rightarrow x^2 = -1$ but all real #'s square to $\geqslant 0$.

+2 **Part E.** $\forall y \in \mathbb{R} : [\exists x \in \mathbb{R} : (xy < 1) \wedge (x > 0)]$ True:

case 1: $y \leq 0$ let $x = 1$. Then $(x > 0) = T$ and $(xy = y \leq 0 < 1) = T$.

case 2: $y > 0$ let $x = \frac{1}{2y}$. Then $(x > 0) = T$ and $\left(xy = \frac{1}{2y} \cdot y = \frac{1}{2} < 1\right) = T$.

+2 **Problem 9.** Is the function $f : \{a, b, c, d\} \to \{u, v, w, x, y\}$ defined by $f(a) = y$, $f(b) = x$, $f(c) = w$, $f(d) = v$ one-to-one? Explain.

Yes: each output comes from a unique input.

+4 **Problem 10A.** Is the function $g : \mathbb{R} \to \mathbb{R}$ defined by $g(x) = 1 - 2x$ one-to-one? Explain.

Yes: let $y \in$ Codomain $(g) = \mathbb{R}$. Then $y = g(x) \Rightarrow y = 1 - 2x \overset{-1}{\Rightarrow} y - 1 = -2x \overset{\div -2}{\Rightarrow} \frac{y-1}{-2} = x$

$\Rightarrow x = \frac{1-y}{2} \in \mathbb{R} \Leftarrow \text{domain}(g)$.

+4 **Problem 10B.** Is the function $h : \mathbb{Z} \to \mathbb{Z}$ defined by $h(x) = 1 - 2x$ onto? Explain.

No: 0 is not an output: $0 = h(x) = 1 - 2x \overset{-1}{\Rightarrow} -1 = -2x \overset{\div -2}{\Rightarrow} x = \frac{1}{2} \notin \mathbb{Z} = \text{domain}(h)$.

So h is not onto.

Problem 11. Suppose that p and q are statements such that $p \Rightarrow q$ is false. ← occurs precisely when $p = T, q = F$.

+2 **Part A.** Find the truth value of $p \vee q$.

$p \vee q \equiv T \vee F = T$

$\boxed{\text{when } p = T, q = F}$

+2 **Part B.** Find the truth value of $\neg p \Rightarrow q$.

$(\neg p \Rightarrow q) \equiv (\neg T \Rightarrow F) \equiv (F \Rightarrow F) \equiv T$

$\boxed{\text{when } p = T, q = F}$

Problem 12. Consider the function $f : \mathbb{N} \to \mathbb{N}$ defined by $f(n) = n^2$.

+3 **Part A.** Is the statement $\forall x \in \mathbb{N} : \underbrace{[\forall y \in \mathbb{N} : (f(x) = f(y)) \Rightarrow (x = y)]}_{f \text{ is one-to-one}}$ true or false? Explain.

True because f is one-to-one on $\mathbb{N}$: $f(n_1) = f(n_2) \Rightarrow n_1^2 = n_2^2 \overset{\text{square root}}{\Rightarrow} n_1 = \pm n_2 \Rightarrow n_1 = n_2$.

n_1, n_2 are not negative

+3 **Part B.** Is the statement $\forall y \in \mathbb{N} : \underbrace{[\exists x \in \mathbb{N} : y = f(x)]}_{f \text{ is onto}}$ true or false? Explain.

False bc f is not onto: 2 is not an output

Since $2 = f(n) = n^2 \overset{\sqrt{\ }}{\Rightarrow} n = \sqrt{2} \notin \mathbb{N} = \text{domain}(f)$.

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Score	22	18	20	60

~~20~~

60 points.